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> W. J. CODY

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## TABLE I

$$
E_{\ell m}=-100 \log _{10}\left\|\frac{C(x)-C_{\ell m}(x)}{C(x)}\right\|_{\infty}
$$

$$
|x| \leq 1.2
$$





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