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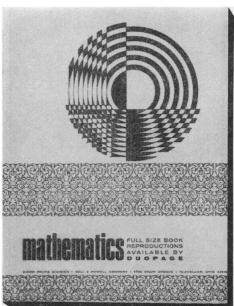
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Mathematics of Computation

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W. J. CODY

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TABLE I

$$E_{gm} = -100 \log_{10} \left| \frac{C(x) - C_{gm}(x)}{C(x)} \right|_{\infty}$$

 $|x| \leq 1.2$

m\	0	1	2 .	3	4	5	6	
01234	60	179 328 457 560	330 519 698 879	505 725 933 1137	698 944 1174 1400 1624	906 1173 1423 1667	1127 1413 1680	1

$$1.2 \le |x| \le 1.6$$

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	35	106	217	354	511	684	94444	***
ĭ		204	363	535	719	915	1120	1
3			501	704 872	1102	1128	1351	
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